

# **Vibration Analysis of Thermo Elastic Micro Beam with Double Porosity Structure**

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## **Abstract**

The present investigation is concerned with vibration analysis of a homogeneous, isotropic thermo elastic micro beam with double porosity structure subjected to sinusoidal pulse heating. Lord-Shulman [1] theory of thermo elasticity with one relaxation time is used to solve the problem. Laplace transform technique has been used to obtain the expressions for lateral deflection, axial stress, axial displacement, volume fraction field and temperature distribution. A numerical inversion technique has been applied to recover the resulting quantities in the physical domain. Variations of axial displacement, axial stress, lateral deflection, volume fraction field and temperature distribution against axial distance are depicted graphically to show the effect of porosity and relaxation time parameters. Some particular cases are also deduced.

**Keywords:** Double porosity, thermo elasticity, Lord-Shulman theory, micro beam, sinusoidal pulse heating

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