Special Multiplicative Operators for the Solution of ODE – Invariants and Representations

Zenonas Navickas[1], Tadas Telksnys[2], Minvydas Ragulskis[2,*]

[1]Department of Applied Mathematics, Kaunas University of Technology, Kaunas, Lithuania.

[2]Research Group for Mathematical and Numerical Analysis of Dynamical Systems, Kaunas University of Technology, Kaunas, Lithuania.

Received 12 July 2014; received in revised form 05 August 2014; accepted 10 September 2014

Abstract

The generalized multiplicative operator of differentiation is introduced in this paper. It is shown that the generalized multiplicative operator can be expressed as a product of two noncommutative but multiplicative exponential operators, though the generalized multiplicative operator is not an exponential operator itself. The generalized multiplicative operator is effectively exploited for the construction of solutions to nonlinear ordinary differential equations through formal transformations of invariants and representations of initial conditions. The concept of the generalized multiplicative operator provides the insight into the algebraic structure of solutions to nonlinear ordinary differential equations which cannot be identified using conventional exponential operators.

Keywords: ordinary differential equation, multiplicative operator, invariant

References

[1] M.V. Demina, N. A. Kudryashov, "Explicit expressions for meromorphic solutions of autonomous nonlinear ordinary differential equations," Communications in Nonlinear Science and Numerical Simulation, vol. 16, pp. 1127-1134, 2011.

[2] A. U. Afuwape, "Frequency domain approach to some third-order nonlinear differential equations," Nonlinear Analysis, vol. 71, pp. 972-978, 2009.

[3] M. El-Gebeily, D. O'Regan, "Existence and boundary behavior for singular nonlinear differential equations with arbitrary boundary conditions," Journal of Mathematical Analysis and Application, vol. 334, pp. 140-156, 2007.

[4] G. Xu, Z. Li, "PDEPtest: a package for the Painleve test of nonlinear partial differential equations," Applied Mathematics and Computation, vol. 169, pp. 1364-1379, 2005.

[5] J. K. Zhou, Differential transform and its applications for electrical circuits, Wuhan: Huarjung University Press, 1986.

[6] A. Arikoglu, I. Ozkol, "Vibration analysis of composite sandwich beams with viscoelastic core by using differential transform method," Composite Structures, vol. 92, pp. 3031-3039, 2010.

[7] G. Adomian, Solving frontier problems of physics: the decomposition method, Kluwer, 1994.

[8] A. Akyuz-Dascioglu, H. Cerdik-Yaslan, "The solution of high-order nonlinear ordinary differential equations by Chebyshev series," Applied Mathematics and Computation, vol. 217, pp. 5658-5666, 2011.

[9] E. W. Kamen, "An operator theory of linear functional differential equations," Journal of Differential Equations, vol. 27, pp. 274-297, 1978.

[10] N. Katz, "A conjecture in the arithmetic theory of differential equations," Bulletin de la Societe Mathematique de France, vol. 110, pp. 515-534, 1982.

* Corresponding author. E-mail address: minvydas.ragulskis@ktu.lt

Tel.: +37069822456; Fax: +37037330446

[11] M. A. H. MacCallum, A. V. Mikhailov, Algebraic theory of differential equations, Cambridge: London Mathematical Society Lecture Note Series (No. 357), 2008.

[12] I. Herrera, "The algebraic theory approach for ordinary differential equations: Highly accurate finite differences," Numerical Methods for Partial Differential Equations, vol. 3, pp. 199-218, 1987.

[13] W. Magnus, "On the exponential solution of differential equations for a linear operator," Communications on Pure and Applied Mathematics, vol. 7, pp. 649-673, 1954.

[14] G. Dattoli, A. M. Mancho, M. Quattromini, A. Torre, "Exponential operators, generalized polynomials and evolution problems," Radiation Physics and Chemistry, vol. 61, pp. 99-108, 2001.

[15] S. H. Leventhal, "An operator compact implicit method of exponential type," Journal of Computational Physics, vol. 46, pp. 138-165, 1982.

[16] P. J. Olver, Classical invariant theory, Cambridge: Cambridge University Press, 1999.

[17] M. Rahula, Vector fields and symmetries (in Russian), Tartu: Tartu University Press, 2004.

[18] N. H. Ibragimov, M. Torrisi, A. Valenti, "Differential invariants of nonlinear equations $v_{tt} = f(x,v_x)v_{xx}+g(x,v_x)$," Communications in Nonlinear Science and Numerical Simulation, vol. 9, pp. 69-80, 2004.

[19] Z. Navickas, "The operator method for solving nonlinear differential equations," Lietuvos matematikos rinkinys, vol. 42, pp. 486-493, 2002.

[20] Z. Navickas, M. Ragulskis, "How far can one go with the Exp-function method?" Applied Mathematics and Computation, vol. 211, pp. 522-530, 2009.

[21] D. Voslamber, "On exponential approximations for the evolution operator," Journal of Mathematical Analysis and Applications, vol. 37, pp. 403-411, 1972.

[22] T. Cluzeau, M. van Hoeij, "A modular algorithm for computing the exponential solution of a linear differential operator," Journal of Symbolic Computation, vol. 38, pp. 1043-1076, 2004.

[23] E. A. Coddington, N. Levison, Theory of ordinary differential equations, New York: McGraw-Hill, 1955.

[24] M. Ragulskis, Z. Navickas, L. Bikulciene, "The solitary solution of the Liouville equation produced by the Exp-function method holds not for all initial conditions," Computers and Mathematics with Applications, vol. 62, pp. 367-382, 2011.